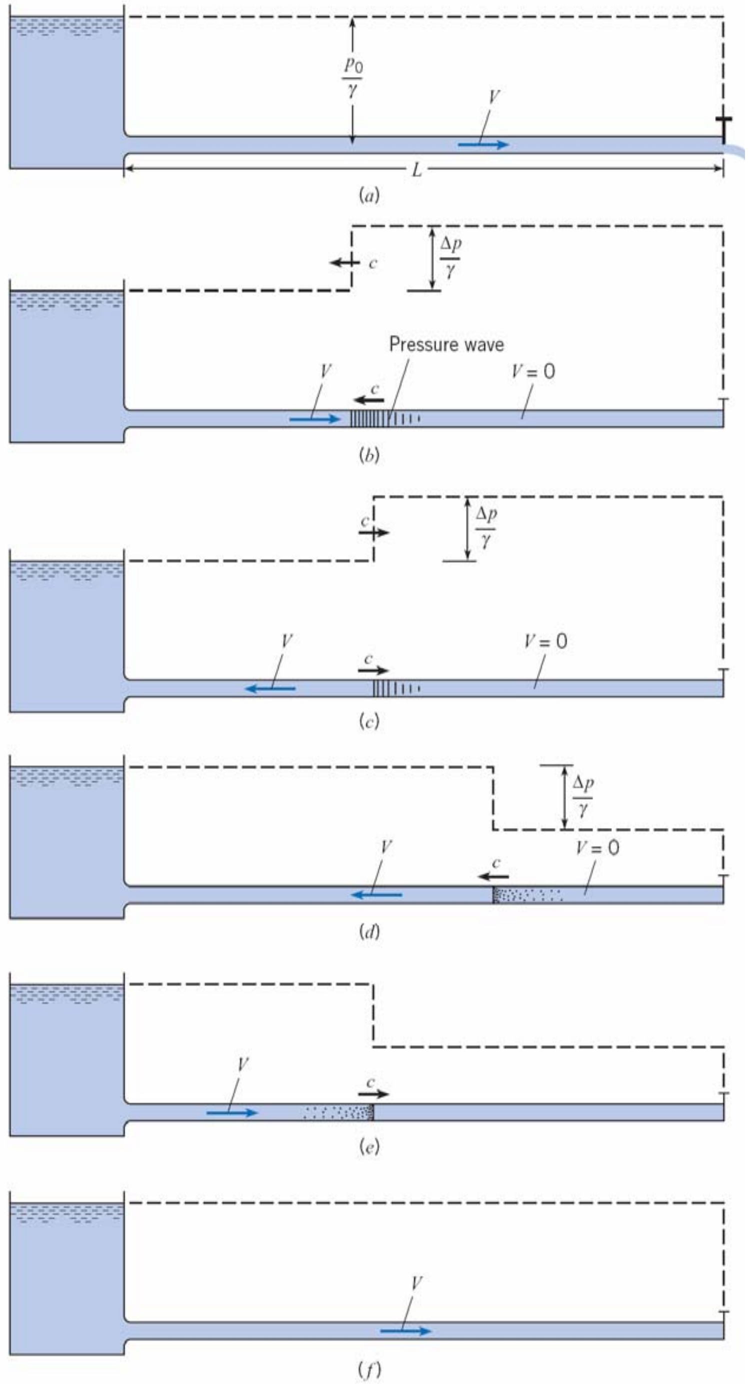
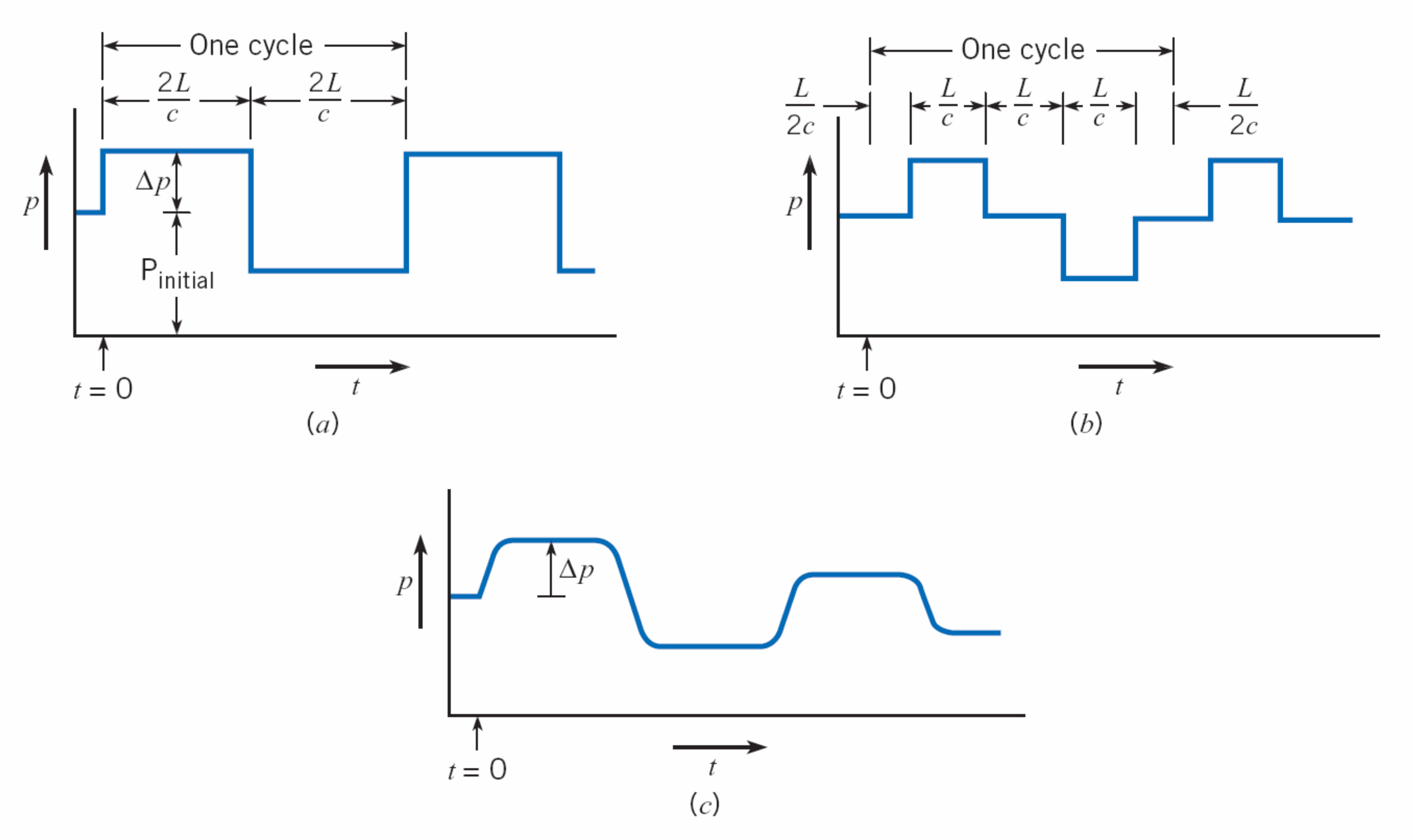
Water Hammer

Water hammer process.

(a) Initial condition.
(b) Condition during time $0 < t < L/c$.
(c) Condition during time $L/c < t < 2L/c$.
(d) Condition during time $2L/c < t < 3L/c$.
(e) Condition during time $2L/c < t < 4L/c$.
(f) Condition at time $t = 4L/c$.





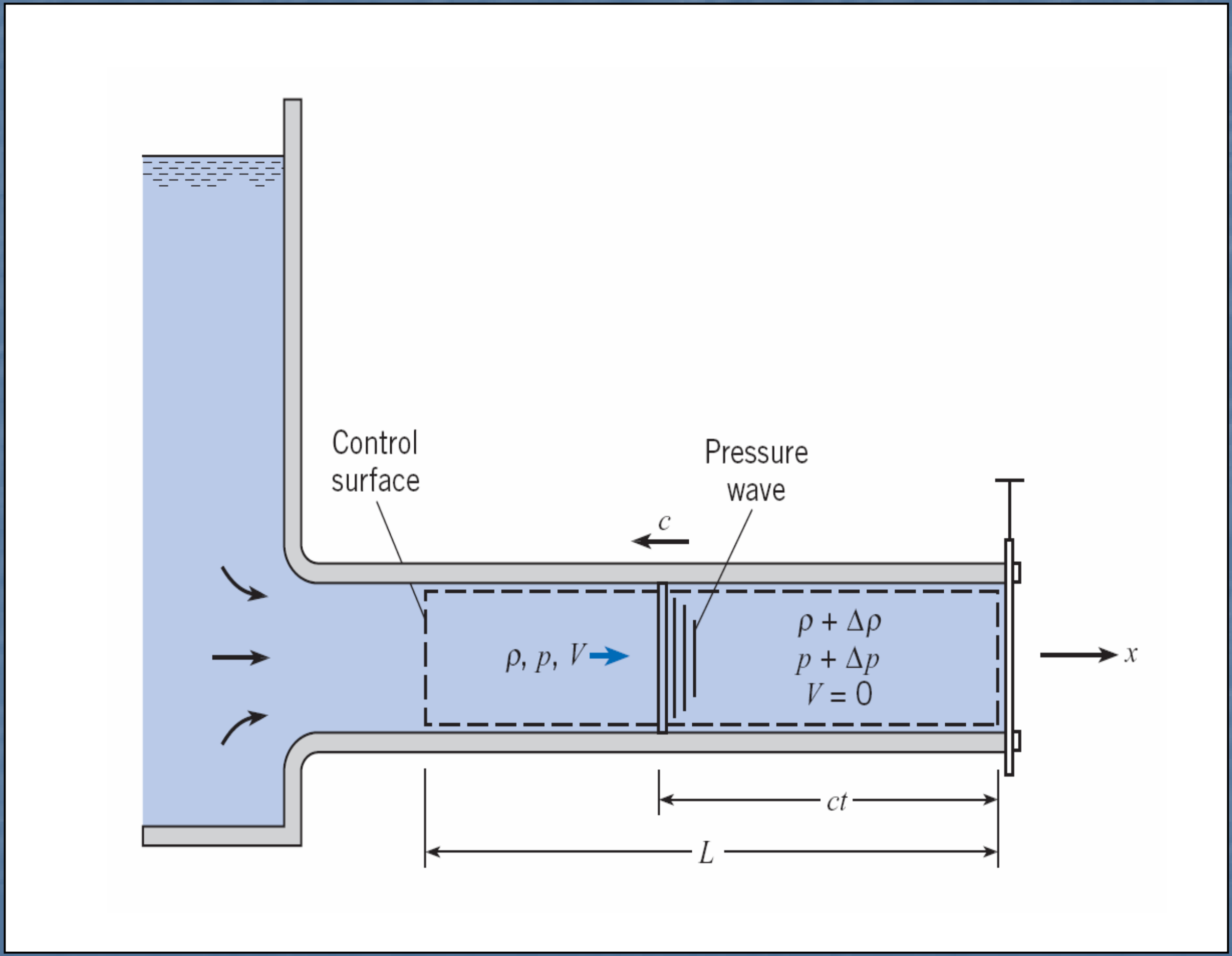
Variation of water-hammer pressure with time at two points in a pipe.

(a) Location: adjacent to valve.

(b) Location: at midpoint of pipe.

(c) Actual variation of pressure near valve.

Magnitude of Water Hammer Pressure and Speed of Pressure Wave



$$-\Delta p\, A = -v\rho c A - \rho A v^2$$

$$\Delta p = \rho v c + \rho v^2$$

The term (ρv^2) can be neglected with respect to the first term because (c) for liquid is much greater than (v) .therefore, the above Eqn. becomes

$$\Delta p = \rho v c$$

Applying the continuity principle to the control volume

$$0 = \frac{d}{dt}\int_{cv} \rho dQ + \sum_{cs} (\dot{m})_{outX} - \sum_{cs} (\dot{m})_{inX}$$

$$0 = -\dot{m}_{in} + \frac{d}{dt}\left[\rho (L - ct) A + (\rho + \Delta\rho) ctA\right]$$

$$0 = -\rho v A + \left[-(\rho c A) + (\rho c A + \Delta\rho c A)\right]$$

$$\frac{\Delta\rho}{\rho} = \frac{v}{c}$$

Hence $c = \dfrac{v}{\Delta\rho/\rho}$

By definition, the gas compressibility $E_v = \dfrac{\Delta p}{\Delta\rho/\rho} = p$

$$c = \frac{vE_v}{\Delta p} = \frac{vE_v}{\rho v c}$$

$$c = \sqrt{\frac{E_v}{\rho}}$$

Moment - of - Momentum Equation

For rotational motion, the angular momentum of a system is given by,

$$\sum M = \frac{d(H)_{sys}}{dt}$$

$M =$ The moment

$H_{sys} =$ angular momentum of the system

Using the Reynolds transport theorem which is

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \int_{cs} b\rho V \bullet dA$$

$$B_{sys} = (H)_{sys} = (mvr)_{sys}$$

$$b = vr$$

$$\frac{d(H)_{sys}}{dt} = \frac{d}{dt}\int_{cv}(v \times r)\rho dQ + \int_{cs}(v \times r)\rho V \bullet dA = \sum M$$

$$\sum M = \frac{d}{dt}\int_{CV}(r \times v)\rho dQ + \sum_{CS} r \times (\dot{m}v)_{out} - \sum_{CS} r \times (\dot{m}v)_{in}$$

MOMENTUM PRINCIPLE

The Momentum Equation for Cartesian Coordinates

X-direction:

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX}$$

Y-direction:

$$\sum F_Y = \frac{d}{dt}\int_{cv} v_Y \rho dQ + \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY}$$

Z-direction:

$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ}$$

Moment - of - Momentum Equation

$$\sum M = \frac{d}{dt}\int_{CV} (r \times v) \rho dQ + \sum_{CS} r \times (\dot{m}v)_{out} - \sum_{CS} r \times (\dot{m}v)_{in}$$

END OF LECTURE (6)